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LETTER TO THE EDITOR

A new type of modulation instability of Stokes waves in the framework of an extended NSE system with mean flow

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Abstract

Stokes waves on the surface of a layer of an ideal fluid are studied. The nonlinear Schrodinger equation (NSE) for the envelope of the first harmonic and the equation for zero harmonic are extended with allowance for full linear dispersion. To investigate modulational instability (MI) of Stokes waves, we derive a quartic equation for the perturbation frequency without the traditional approximation for the motion of mean current with a group speed on the frequency of fast filling. The interaction of the four roots of this equation is shown to result in the occurrence of MI bands not described by the NSE. The analysis of the obtained expressions demonstrates that the limit $kh = 1.363$ (where h is the fluid depth and k is the wave number) found by Benjamin and Feir (and also by Whitham and then by Hasimoto and Ono) for the transition between the states of modulationally stable and unstable liquid is valid only in the limiting case of small amplitudes of unperturbed waves and small wave numbers of the perturbation wave.

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Modulational instability (MI) is the growth of modulations of steady oscillating nonlinear waves under the action of small harmonic perturbations. This is a widely observed phenomenon related to wave propagation in various media. In the case of gravity waves on the fluid surface, studying the modulational instability is of high importance in view of the conjecture that the occurrence of the so-called freak waves in open seas is a result of long-term MI evolution [1]. The bibliography on 'freak waves' and 'modulational instability' can be found by using a web search engine.

When an impulse of rapidly oscillating waves propagates on the surface of an ideal fluid (Stokes waves), the amplitude A of the envelope of the first harmonic of the free surface profile $\eta = \frac{1}{2}A\exp(\mathrm{i}(kx - \omega t)) + \text{c.c.}$ and the amplitude Ψ of the zero harmonic of the velocity potential (long waves, mean flow) satisfy the system of coupled equations in the approximation $O(\varepsilon^3)$ [2–4]

$$\mathrm{i}\left(\frac{\partial A}{\partial t} + c_g\frac{\partial A}{\partial x}\right) + p\frac{\partial^2 A}{\partial x^2} + \widetilde{q}A^2\overline{A} - \left(k\frac{\partial \Psi}{\partial x} + \mu\frac{\partial \Psi}{\partial t}\right)A = 0, \tag{1}$$

$$\frac{\partial^2 \Psi}{\partial t^2} - c_0^2\frac{\partial^2 \Psi}{\partial x^2} - \nu\frac{\partial}{\partial x}A\overline{A} = 0, \tag{2}$$

where ε is a small parameter that characterizes the smallness of the amplitudes A and Ψ and the slowness of their change in time and space. Interrelation of the first harmonic and mean flow is described by the last terms of these equations.

Here we denote

$$\begin{aligned}
&c_g \equiv \frac{\partial\omega}{\partial k} = \frac{\omega}{k}C_g, && C_g = \frac{1}{2} + \frac{1-\sigma^2}{2\sigma}kh,\\
&c_0 \equiv \sqrt{gh} = \frac{\omega}{k}C_0, && C_0 = \sqrt{\frac{kh}{\sigma}},\\
&\sigma = \tanh kh\ , && p \equiv \frac{1}{2}\frac{\partial^2\omega}{\partial k^2} = \frac{\omega}{k^2}P,\\
&P = \frac{1}{8\sigma^2}((\sigma^2-1)(3\sigma^2+1)k^2h^2 - 2\sigma(\sigma^2-1)kh - \sigma^2), &&\\
&\widetilde{q} = \omega k^2\widetilde{Q}, && \widetilde{Q} = \frac{1}{16\sigma^4}(2\sigma^6 - 13\sigma^4 + 12\sigma^2 - 9),\\
&\mu = \frac{k^2}{\omega}M, && M = \frac{1}{2}(\sigma^2 - 1\ ),\\
&\nu = \frac{\omega^3}{k}N, && N = \frac{1}{2\sigma^2}[1 - MC_g],
\end{aligned}$$

$\omega = \sqrt{gk\sigma}$ is the carrier wave frequency and g is the acceleration due to gravity.

System (1)–(2) is a model of the interaction of long and short waves. It is used in various physical problems of nonlinear optics and plasma physics and arises in studies of water waves [5, 6]. In hydrodynamics, it is called the NSE system with the equation for mean flow (NSE-mean), and it is also known as the Benney–Roskes system [3] or the Davey–Stewartson-type system.

One of the possible applications of this system is the problem of modulational stability/instability described by equation (1) for the fundamental harmonic of rapidly oscillating waves with the wave length $2\pi/k$ under the action of a small harmonic perturbation with the wavelength $2\pi/\varkappa$. It is well known [2, 7–9] that, when $\varkappa$ is much smaller than k, gravity waves are stable for $kh < 1.363$. For $kh > 1.363$, two-dimensional Stokes waves are unstable in some range $0 < \varkappa < \varkappa_0(kh)$ and stable again when $\varkappa > \varkappa_0(kh)$. This is a well-known fact, and thresholds like $kh = 1.363$ were obtained for some other wave types [10]. Instability implies that perturbations grow until they are stopped by the stabilization mechanism due to the increasing effect of nonlinearity and dispersion.

Note that all the above-listed results were obtained with the assumption of small $\varkappa$ (the so-called Benjamin–Feir instability). On the other hand, the problem on the MI of Stokes waves is considered not only for small wave vectors of harmonic perturbation. For example, the perturbation wave numbers $\varkappa$ were arbitrary in the numerical studies of class II MI [11, 12] and in the studies based on Zakharov's equations [13].

If we raise the question of the modulational stability of Stokes waves when the wavelength of harmonic perturbation is of the same order as the wavelength of the carrying wave, our procedure should be similar to that of Hasimoto and Ono, but it should differ in two essential points:

Point 1. Departure from the widely accepted assumption that a potential of zero harmonic depends on x and t in combination $x - c_g t$, where $c_g = \frac{\partial \omega}{\partial k}$ is the group velocity on the frequency of the first harmonic A [14].

Point 2. The generalization of system (1)–(2) should be made by taking into account all the linear terms.

Here we give a more detailed explanation.

Point 1. To depart from the approximation of small κ, one should depart from the assumption that the zero harmonic of the potential depends on x and t in a combination $x - c_g t$. This assumption is especially met when NSE is derived from system (1)–(2) in order to reduce the system to one equation. Some authors call it the transition into the frame that moves with the group velocity c_g of linear waves, and others argue with forcing action of the third term in (2) which really evolves in time with the group velocity c_g.

In the specific case of not small wave numbers $\varkappa$, a group velocity c_g of pure gravity waves on a surface of a fluid comes nearer to a velocity long wave c_0 and the assumption about driving a zero harmonics with a group velocity c_g becomes justified, i.e. the substitution $\frac{\partial \Psi}{\partial t} = -c_g \frac{\partial \Psi}{\partial x}$ into equation (2) for small $\varkappa$ can be done. Note that, as proved in [15], a zero harmonic Ψ depending on x and t only through $x - c_g t$ is not a hypothesis, but it is a property system involving the equations originating from its derivation, which for us concerns the case of long wavelength modulation instability.

However, in the case of arbitrary wave numbers $\varkappa$ of the perturbation wave, the assumption may be incorrect. The groundlessness of the above-mentioned assumption is discussed in [16, 17]. In [18], it is shown that originating at replacement of $\xi = x - c_g t$ additional items is necessary to take into account in the next approximation on ε at deriving the NSE of the fourth order. The possibility for simplification of equation (2) and for the closure of system (1)–(2) was considered in the series of papers [15, 16, 19, 20].

The departure from the above-mentioned assumption means impossibility of replacement $\frac{\partial \Psi}{\partial t} = -c_g \frac{\partial \Psi}{\partial x}$, and it means impossibility of closure of the system (1)–(2) in NSE. That puts in doubt the invariance of the value $kh = 1.363$, since upon application of this replacement (for example in [2]), equation (2) is integrated, a derivative $\frac{\partial \Psi}{\partial x}$ from (2) substitutes in (1), which then transforms into the usual NSE

$$\mathrm{i}\left(\frac{\partial A}{\partial t} + c_g \frac{\partial A}{\partial x}\right) + p\frac{\partial^2 A}{\partial x^2} + qA^2\overline{A} = 0, \tag{3}$$

where

$$q = \widetilde{q} + \frac{k^2}{\omega^3}\frac{2\sigma^2 \nu^2}{c_0^2 - c_g^2} = \omega k^2\left(\widetilde{Q} + \frac{2\sigma^2 N^2}{C_0^2 - C_g^2}\right). \tag{4}$$

As q changes sign from positive to negative at $kh = 1.363$ and as NSE has soliton solutions under zero boundary conditions at $pq > 0$, and $p < 0$ for all kh, NSE (3) cannot have soliton solutions if $kh < 1.363$. That Stokes waves are stable at $kh < 1.363$ follows also from the expression for increment MN obtained in [2] on the basis (3)

$$\mathrm{Im}\,\Omega = \varkappa\left(2pqA_0^2 - \varkappa^2 p^2\right)^{\frac{1}{2}}, \tag{5}$$

containing the same expression pq.

Point 2. To reconstruct the amplitude equations one should use several different small parameters. In nonconservative media taking account of terms of the basic order on dispersion and nonlinearity the natural small parameter describing proximity of a system from a critical state is usually used. Introduction of the second small parameter is necessary taking account of terms of higher order [21] (equation (3.2) and after). In a conservative system introduction of two independent small parameters which characterize the smallness of the amplitudes and the slowness of their change in space was produced, for example, in [22] for Stokes waves on a surface of a deep fluid. However, when these methods are used strictly, high-order linear terms should be taken into account together with high-order nonlinear terms. Since taking into account nonlinear terms to high order is a complicated technical problem, high-order linear terms are also neglected and their contribution is thereby underestimated. For example, the instability region on the plane of two wave vector components turns out to be non-closed when only the parabolic dispersion is taken into account (ordinary NLS). This corresponds to the unbounded increase of energy as the wave vectors of the disturbing wave increase. On the other hand, taking into account the cubic dispersion (while freezing the order of nonlinearity) results in the corresponding turn of the instability curve, and taking into account all the linear terms results in the closed curve (the Philips eight). In our study, taking into account all the linear terms in both equations leads to the solid curves in figures 1 and 2 instead of the dashed curves which are obtained when only the first linear term is taken into account. New dispersion curves (due to the above-mentioned point 1) attract each other in the region which is shifted to smaller wave numbers as compared to the case when only several linear terms are considered [14].

In other words, one has to go beyond the spectrally narrow approximation of the NSE type when considering wide-band wave trains. The linear terms are all naturally taken into account in the Zakharov equations [23, 24] and in the Benney–Luke–Milewski equations [25]. So, the generalization of system (1)–(2) is made here by adding to it all linear terms. Such an addition for system (1)–(2) was produced in [26] for a special case of an indefinitely deep fluid, and at the rejection of the time derivative in (2).

The account of all linear terms in (1)–(2) within a departure from connection x and t in a combination $x - c_g t$ at description of MI of Stokes waves for fluid layer is the basic purpose of the given work.

For the first equation of system (1)–(2) introduction of all linear items will be carried out by replacement of the second and third items by the infinite sum

$$\pounds A = \mathrm{i}c_g\frac{\partial A}{\partial x} + p\frac{\partial^2 A}{\partial x^2} - \mathrm{i}\frac{1}{6}\frac{\partial^3\omega}{\partial k^3}\frac{\partial^3 A}{\partial x^3} - \frac{1}{24}\frac{\partial^4\omega}{\partial k^4}\frac{\partial^4 A}{\partial x^4} + \cdots.$$

For adding the remaining linear terms to the linear item $c_0^2\frac{\partial^2\Psi}{\partial x^2}$ of the second equation of system (1)–(2) we shall understand that equation (2) at the deduction (1)–(2) by the method of multiple scales [4] is the so-called compatibility condition. According to the alternative of Fredholm in approximation (ε^3) it could be presented as [16]

$$\frac{\partial^2\,\Phi}{\partial t^2} + g\int_{-h}^{0}\frac{\partial^2\,\Phi}{\partial x^2}\,\mathrm{d}z - \nu\frac{\partial}{\partial x}A\overline{A} = 0, \tag{6}$$

and it is important that with prolongation of the asymptotic procedure only nonlinear terms are accumulated, i.e. the complete account of the linear terms (2) contains in the second term (6). Here Φ is the whole potential zero harmonic, while Ψ is the potential of zero harmonics only of the first order of asymptotic procedure of method of multiple scales [4]:

$$\Phi = \Psi + \varepsilon\Psi + \varepsilon^2\frac{\partial^2\Psi}{\partial x^2}(z+h)^2.$$

After transition to the new function Φ instead of Ψ, the extended system (1)–(2) looks like

$$\mathrm{i}\frac{\partial A}{\partial t} + \pounds A + \widetilde{q}A^2\overline{A} - \left(k\frac{\partial \Phi}{\partial x} + \mu\frac{\partial \Phi}{\partial t}\right)\bigg|_{z=0} A = 0, \tag{7}$$

$$\frac{\partial^2 \Phi}{\partial t^2} + g\int_{-h}^{0}\frac{\partial^2 \Phi}{\partial x^2}\,\mathrm{d}z - \nu\frac{\partial}{\partial x}A\overline{A} = 0. \tag{8}$$

System (7)–(8) has the homogeneous solution on x that describes the unperturbed Stokes wave

$$A = A_0\,\mathrm{e}^{\mathrm{i}\alpha t}, \qquad \Phi = 0,$$

where $\alpha = \widetilde{q}A_0^2$ and the amplitude A_0 does not depend on coordinates and time. Let us introduce a perturbation (a is the complex one, and φ is a real one)

$$A = (A_0 + \epsilon a)\,\mathrm{e}^{\mathrm{i}\alpha t}, \qquad \Phi = \epsilon\varphi.$$

System (7)–(8), being linearized in ϵ, looks like

$$\begin{aligned}
&\mathrm{i}\frac{\partial a}{\partial t} + \pounds a + \widetilde{q}A_0^2(a+\overline{a}) - \left(k\frac{\partial \varphi}{\partial x} + \mu\frac{\partial \varphi}{\partial t}\right)\bigg|_{z=0} A_0 = 0,\\
&\frac{\partial^2 \varphi}{\partial t^2} + g\int_{-h}^{0}\frac{\partial^2 \varphi}{\partial x^2}\,\mathrm{d}z - \nu A_0\left(\frac{\partial a}{\partial x} + \frac{\partial \overline{a}}{\partial x}\right) = 0.
\end{aligned} \tag{9}$$

We represent the solution of the linear system of integral differential equations (9) as

$$a = a_0\,\mathrm{e}^{\mathrm{i}\theta} + b_0\,\mathrm{e}^{-\mathrm{i}\theta}, \qquad \theta = \varkappa x - \Omega t$$

$$\phi = (\psi_1\,\mathrm{e}^{\mathrm{i}\theta} + \psi_2\,\mathrm{e}^{-\mathrm{i}\theta})\frac{\cosh\varkappa(z+h)}{\cosh\varkappa h}.$$

Substituting it into (9), we obtain the linear system of algebraic equations

$$\begin{aligned}
&\big(\Omega - \omega(k+\varkappa) + \omega(k) + \widetilde{q}A_0^2\big)a_0 + \widetilde{q}A_0^2 b_0 + \mathrm{i}(\mu\Omega - k\varkappa)A_0\psi_1 = 0\\
&\big(\Omega + \omega(k-\varkappa) - \omega(k) - \widetilde{q}A_0^2\big)b_0 - \widetilde{q}A_0^2 a_0 + \mathrm{i}(\mu\Omega - k\varkappa)A_0\psi_2 = 0\\
&\mathrm{i}\nu\varkappa A_0(a_0+b_0) + (\Omega^2 - \omega^2(\varkappa))\psi_1 = 0\\
&\mathrm{i}\nu\varkappa A_0(a_0+b_0) - (\Omega^2 - \omega^2(\varkappa))\psi_2 = 0,
\end{aligned}$$

Eliminating ψ_1 and ψ_2 we obtain

$$\big[\Omega - \omega(k+\varkappa) + \omega(k) + q(\Omega)A_0^2\big]a_0 + q(\Omega)A_0^2 b_0 = 0$$

$$\big[\Omega + \omega(k-\varkappa) - \omega(k) - q(\Omega)A_0^2\big]b_0 - q(\Omega)A_0^2 a_0 = 0,$$

where

$$q(\Omega) = \widetilde{q} + \frac{k\varkappa - \mu\Omega}{\omega^2(\varkappa) - \Omega^2}\nu\varkappa. \tag{10}$$

Equating the determinant to zero gives the equation for the perturbation frequency Ω:

$$(\Omega - \delta)^2 = \Delta^2 - 2q(\Omega)A_0^2\Delta, \tag{11}$$

where

$$\Delta = \tfrac{1}{2}[\omega(k+\varkappa) + \omega(k-\varkappa)] - \omega(k),$$

$$\delta = \tfrac{1}{2}[\omega(k+\varkappa) - \omega(k-\varkappa)].$$

Taking the account of the only two first terms of linear dispersion on the carrier frequency in (1) and the first term of linear dispersion of long waves in (2) we have

$$\delta \to c_g\varkappa, \quad \Delta \to p\varkappa^2, \quad \omega(\varkappa) \to c_0\varkappa. \tag{12}$$

Equation (11) can be presented as

$$[(\Omega-(\delta-\Delta)][(\Omega-(\delta+\Delta)][\omega(\varkappa)-\Omega][\omega(\varkappa)+\Omega] \\ = -2(\widetilde{q}(\omega^2(\varkappa)-\Omega^2)+(k\varkappa-\mu\Omega)\nu\varkappa)A_0^2\Delta, \tag{13}$$

which illustrates the interaction of four roots of the dispersion equation (11). At small nonlinearity in the right-hand part of (13) these four roots correspond to branches 1, 2, 3, 4 in figures 1 and 2.

Taking into account (10), we get

$$\Omega^4-2\delta\Omega^3-\left(\Delta^2-2\widetilde{q}A_0^2\Delta+\omega^2(\varkappa)-\delta^2\right)\Omega^2+2\left(\delta\omega^2(\varkappa)+\varkappa\mu\nu A_0^2\Delta\right)\Omega \\ +\left(\left(\Delta^2-2\widetilde{q}A_0^2\Delta-\delta^2\right)\omega^2(\varkappa)-2k\varkappa^2\nu A_0^2\Delta\right)=0. \tag{14}$$

Let us make the parameters of the problem dimensionless:

$$\widehat{\Omega}=\frac{\Omega}{\omega},\qquad \widehat{\Delta}=\frac{\Delta}{\omega},\qquad \widehat{\delta}=\frac{\delta}{\omega},\qquad \widehat{\omega}(\widehat{\varkappa})=\frac{\omega(\varkappa)}{\omega},\qquad \widehat{\varkappa}=\frac{\varkappa}{k},\qquad \widehat{A}_0=kA_0.$$

The result of numerical solution of the normalized equation (14) is shown in figures 1 and 2. Two new factors taken into account in this work result in the following peculiarities.

(1) The departure from the assumption that the potential of zero harmonic depends on x and t in the combination $x-c_gt$ follows into occurrence of the third and fourth factors in (13) with the dispersion relation, common for long waves and therefore (13) describes interaction of not only 1 and 2 waves (at small $\varkappa$—Benjamin–Feir instability), but also, for example, of branches 1 and 3—of one more instability. In this connection let us remark that the equation similar to (11) for the characteristic velocity $\Omega/\varkappa$ has been obtained by the method of the averaged Lagrangian [8] (equation (56)). To simplify the analysis of the fourth-order equation solutions on Ω in the assumption of small A_0 and $\varkappa$, the second term in [8] was neglected and into the third term the relation $\Omega/\varkappa=c_g$ was substituted. The quadratic equation obtained in such a way has a solution whose imaginary part exists only at small wave vectors of perturbation $\varkappa$ and does not exist for any $\varkappa$, if $kh<1.363$. Thus the behaviour analysis of solutions of the secular equation demonstrated in [8] concerns in essence only the case of infinitesimal A_0 and small $\varkappa$ and cannot be applied to the explanation of MI occurrence in place of intersection of roots 1 and 3.

(2) Taking account of additional terms of a linear dispersion in the equation for enveloping and the equation of zero harmonic follows into the exact curvature of four curves instead of their asymptotical behaviour (12), shown by the dashed lines in figures 1 and 2. In particular, this course of curves leads to one more intersection of roots 1 and 2, which is not predicted by the NSE and corresponds in the linear case to taking full account of dispersion to long edges of the eight of Phillips. The equation of the fourth order (the generalized on 3D—a case and including the surface tension) has been obtained by a variation method in [27], but without taking into account all linear dispersion. Therefore, it cannot be applied for explanation of the MI band at that point where curves 1 and 2 intersect a second time and their asymptotes shown by the dotted line intersect only at small $\varkappa$, i.e. they describe only MI of Benjamin–Feir.

The equation of the fourth order for Ω taking complete account of a linear dispersion has been obtained in [28] from Zakharov equations in the ε^3 approximation. For deriving MI criteria it has been simplified with the assumption of small A_0 into the second-order equation for a small number of deviation Ω from the curve $\Delta=0$, but has not been solved numerically.

The peculiarity of the given work is that the secular equation of the fourth order close to that obtained by the Hamilton method in [28] is deduced from the NSE system generalized here with mean flow. Numerical calculations show that the Stokes waves have instability in the region $\varkappa\simeq k$ in addition to the Benjamin–Feir instability at small $\varkappa$.

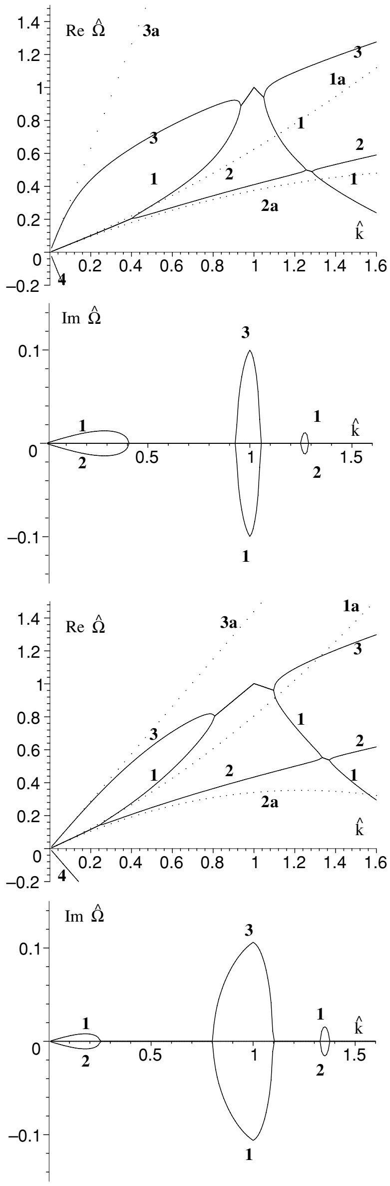

Figure 1. Re $\widehat{\Omega}$, Im $\widehat{\Omega}$ for $kh = 10$ and $kh = 2$ at $\widehat{A}_0 = 0.2$.

We did not investigate the long-term evolution of unstable waves. It may differ from the long-term development of Benjamin–Feir instability, since it is described by system (7)–(8) rather than the NSE. Note that the simplified version system (1)–(2) is solved by the method of the inverse scattering problem [29].

When $kh = 1.363$, the Benjamin–Feir instability disappears, while the MI band we found at $\kappa \simeq k$ does not vanish.

For the determination of kh and $\widehat{\varkappa}$, at which equation (14) has the imaginary part which is distinct from zero the expression for frequency of perturbation $\widehat{\Omega} = \mathrm{Re}\,\widehat{\Omega} + \mathrm{i}\,\mathrm{Im}\,\widehat{\Omega}$ has been

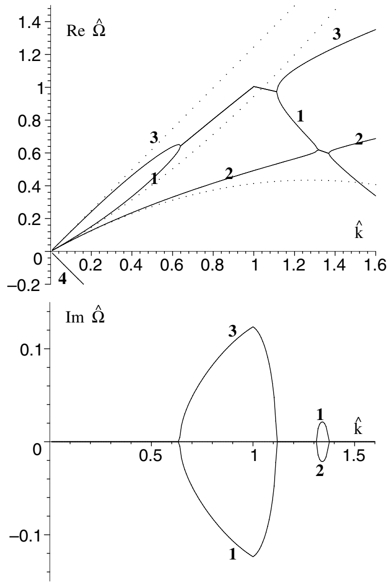

Figure 2. Re $\widehat{\Omega}$, Im $\widehat{\Omega}$ for $kh = 1.363$ at $\widehat{A}_0 = 0.2$.

substituted into (14). After separations of real and imaginary parts the resultant of the system of two nonlinear algebraic equations gives the obtained equation for $(\mathrm{Im}\,\widehat{\Omega})^2$. Retaining in it the items with degrees not higher than $\widehat{A}_0^2$, as the theory is weakly nonlinear, it is possible to obtain for $(\mathrm{Im}\,\widehat{\Omega})^2$ the basic term and the first correction on $\widehat{A}_0^2$. The basic term is

$$(\mathrm{Im}\,\widehat{\Omega})^2 = -\left(\widehat{\Delta}^2 - 2\widehat{\Delta}\left(\widetilde{Q} + \frac{2\sigma^2 N^2}{\widehat{\omega}^2(\widehat{\varkappa}) - \widehat{\delta}^2}\right)\right). \qquad (15)$$

Tabulation of the expression for $\widehat{\Delta}$ in association from kh and $\varkappa$ shows that it could be with both signs that differs from the approximation taking account of only first terms of linear dispersion, when $\widehat{\Delta} \to P\widehat{\varkappa}^2$ and $P < 0$ for all kh.

Conclusion. By taking into account points (1) and (2) we obtained the quartic equation for the frequency of the perturbation wave, which depends on wave number $\varkappa$. Numerical analysis of the imaginary part of four solutions to this equation shows that there is MI at $\hat{\varkappa} \simeq 1$ in addition to the BF instability at small $\hat{\varkappa}$. This instability does not disappear at $kh < 1.363$, in contrast to BF MI. The description of this instability is related to taking into account the effect of the mean flow more thoroughly. This fact suggests that the expected MI can evolve into wave structures intermediate to NSE envelope solitons and solitary waves on shallow water.

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